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A Half-Space Problem in the Fractional Order Theory of Thermoelastic Diffusion

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In this work, a general solution to the field equations of generalized thermoelastic diffusion in a half-space is considered in the context of fractional order theory of thermoelastic diffusion. The bounding surface of the half-space is taken to be traction free and subjected to a time dependent thermal shock while the chemical potential is assumed to be a known function of time. Laplace transform techniques are used. The analytical solution in the transform domain is obtained by using the eigenvalue approach. Numerical results for the temperature, the displacement, the concentration, stress and chemical potential represented graphically.

Keywords: Fractional Order, Thermoelastic Diffusion, Laplace Transform, Eigenvalue Approach.

RESEARCH ARTICLE

1. INTRODUCTION

Biot[1] formulated the theory of coupled thermoelasticity to eliminate the paradox inherent in the classical uncoupled theory that elastic changes have no effect on the temperature. The heat equations for both theories, however, are of the diffusion type predicting infinite speeds of propagation for heat waves contrary to physical observations. The first of such modeling is the extended thermoelasticity theory (LS) of Lord and Shulman,[2] who introduced the concept of thermal relaxation time into the classical Fourier law of heat conduction. Subsequently, modifying the stress versus strain relationship as well as the entropy relationship with relaxation time, Green and Lindsay[3] proposed the temperature rate dependent thermoelasticity (GL) theory. The theory was extended for anisotropic body by Dhaliwal and Sherief.[4] In the last twenty years,[5–11] have considered different problems by the generalized thermoelasticity theories. A survey article of representative theories in the range of generalized thermoelasticity is given by Hetnarski and Ignaczak.[12] Recently,[13–49] have considered different problems by using the generalized thermoelasticity theories.

During recent years, several interesting models have been developed by using fractional calculus to study the physical processes particularly in the area of heat conduction, diffusion, mechanics of solids, electricity etc. Caputo and Mainardi[50, 51] and Caputo[52] have employed the fractional order derivatives for the description of viscoelastic materials and have established the connection between fractional derivatives and the theory of linear viscoelasticity and found a good agreement with the experimental results. Sherief et al.,[53] Youssef[54] and Ezzat,[55] introduced new models of thermoelasticity using a fractional heat conduction equation.

These days, oil companies are interested in the process of thermodiffusion for more efficient extraction of oil from oil deposits. Diffusion can be defined, as the migration of particles from regions of high concentration to regions of lower concentration. Thermodiffusion in an elastic solid is due to coupling of the fields of temperature, mass diffusion and that of strain. The theory of thermoelastic diffusion is developed by Nowacki.[56–58] In this theory, the coupled thermoelastic model is used. Recently, Sherief et al.[59] developed the theory of generalized thermoelastic diffusion that predicts finite speeds of propagation for thermoelastic and diffusive waves. Ezzat and Fayik[60] presented a new theory of thermodiffusion in elastic solids is derived using the methodology of fractional calculus.

The present investigation is devoted to study the fractional order thermoelastic diffusion in a half-space by using Laplace transform and eigenvalue approach. Numerical results for all variables in physical space–time domain are represented graphically.

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